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Toward a Different Portfolio Weighting and Diversification

Charles J. Higgins[1], PhD

Abstract

An examination of portfolio weightings including equal weightings, weightings adjusted for standard deviation, minimum variance portfolio weightings, and risk-return preferential weightings

Keywords: portfolio, weighting, diversification, and efficient frontier

An investment portfolio has Weightings calculated from

1) $W_i = \$_i/{}_k\Sigma^N\$_k$

Where $\$_i$ is the amount invested in each security and where here ${}_i\Sigma^N$ means a summation indexed by i from 1 to N securities, thus:

2) ${}_i\Sigma^N W_i = 1$.

The portfolio Return is:

3) $R_p = {}_i\Sigma^N W_i R_i$

Given that each security return

4) $R_i \equiv (P_{i,1} - P_{i,0} + D_{i,1})/P_{i,0}$

For the terminal and initial Prices and Dividend(s) for each security i. The risk of the portfolio variance is measured by:

5) $\sigma_p{}^2 = {}_i\Sigma^N{}_j\Sigma^N W_i W_j \sigma_{ij}$

Where σ_{ij} is the covariance between each security return.

Often the weightings for a naïve portfolio (see Elton, Gruber, Brown, and Goetzmann) specify that $W_i = 1/N$. Separating the terms where $i = j$ and $i \neq j$ there are N variance terms of $\sigma_i{}^2 = \sigma_{ii}$ and diversification results in elimination of the variance component (or the idiosyncratic risk of a portfolio) when N becomes large, or:

6) $\sigma_p{}^2 = (N/N^2)\underline{\sigma}_i{}^2 + ([N^2\text{-}N]/N^2)\underline{\sigma}_{ij} = (1/N)\underline{\sigma}_i{}^2 + ([N\text{-}1]/N)\underline{\sigma}_{ij}$

Where $\underline{\sigma}_i{}^2$ and $\underline{\sigma}_{ij}$ are the average portfolio variance and covariance.

Now consider a different weighting for minimizing risk with a larger weighting à la Sharpe (see Investopedia and Wikipedia) for securities with a smaller standard deviation and vice versa so that each security would have the same weighted proportional effect upon the portfolio from:

[1]Department of Finance, Loyola Marymount Univ.; Los Angeles, CA 90045, charles.higgins@lmu.edu

7) $W_i = (1/\sigma_i)/{}_k\Sigma^N(1/\sigma_k)$.

If such weightings were to be made, the portfolio variance becomes quite different:

8) $\sigma_p{}^2 = {}_i\Sigma^N{}_j\Sigma^N([1/\sigma_i]/{}_k\Sigma^N[1/\sigma_k])([1/\sigma_j]/{}_k\Sigma^N[1/\sigma_k])\varrho_{ij}\sigma_i\sigma_i$

with correlation ϱ_{ij} and where $\sigma_{ij} \equiv \varrho_{ij}\sigma_i\sigma_j$. But that directly simplifies to:

9). $\sigma_p{}^2 = {}_i\Sigma^N{}_j\Sigma^N(1/{}_k\Sigma^N[1/\sigma_k])^2\varrho_{ij} = 1/({}_k\Sigma^N[1/\sigma_k])^2{}_i\Sigma^N{}_j\Sigma^N\varrho_{ij}$

Consider say a four security portfolio with respective annual standard deviations of .1, .2, .3, and .4 noting that generally security standard deviations increase by the square root of time so that with some 256 trading days per year that the annual standard deviation is about 16 times larger than that of the daily standard deviation. The reciprocals of the four standard deviations are respectively 10, 5, 3.33, and 2.5 and total 20.833. Thus the portfolio weightings respectively would be .48, .24, .16, and .12 totaling 1. For this example there would be 16 (4^2) correlations each multiplied by $1/20.833^2 = .002305$ which if each and every correlation equaled 1 the portfolio variance would be .03688 and thus have a maximum standard deviation of .192. Confirmation is found in the special case when $\varrho_{ij} = 1$ where $\sigma_p = {}_i\Sigma^N W_i\sigma_i$ or here also .192 = .48x.1 + .24x.2 + .16x.3 + .12x.4.

Given $P_t = P_{t-1}(1-\sigma)$ at one extreme and that $P \geq 0$ requires computationally that $\sigma \leq 1$ (and which is generally observed) and when contrasted with $-1 \leq \varrho_{ij} \leq 1$ allows a strong confidence that equation 9) holds rigorously. It follows that one would expect that $1/{}_k\Sigma^N(1/\sigma_k) < 1$ and more so for $(1/{}_k\Sigma^N[1/\sigma_k])^2 < 1$. Therefore one would expect as additional securities are added to a portfolio that the portfolio variance would likely decease. Again when i = j the idiosyncratic risk is diminished as the number of securities is increased to the portfolio noting that here $\varrho_{ii} = 1$ and thus for this component is $N/({}_k\Sigma^N[1/\sigma_k])^2$ and that for each additional increase of N is less than the increase of $({}_k\Sigma^N[1/\sigma_k])^2$ when reminded that $\sigma_i < 1$.

There is a different approach to portfolio weightings where a consideration for return is also included. Given a feasible set of portfolio choices, one would prefer weightings along the uppermost Efficient Frontier while also considering one's risk-return preference:

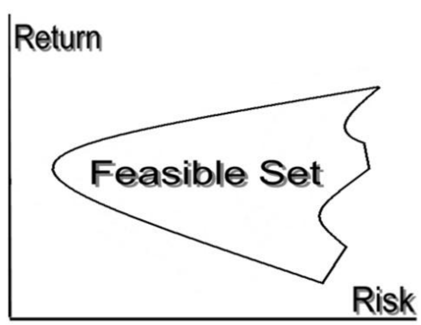

Fig. 1 Feasible Set in Risk-Return Space

The addition of the Capital Market Line usually follows often with a discussion of separating the Market and Risk-Free choices:

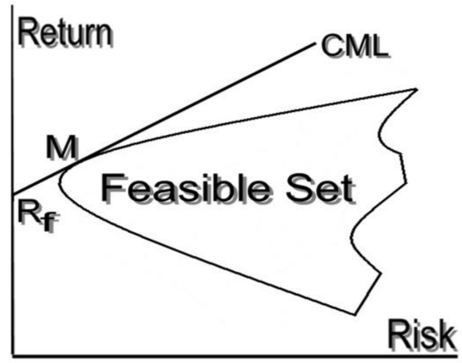

Fig. 2 Feasible Set with the Capital Market Line

However a portfolio weighting along the Efficient Frontier would be available. In an extreme example (which follows) where the securities have negative correlations, the worst security in terms of both risk and return (here labeled e) is still included in a portfolio that is along the Efficient Frontier:

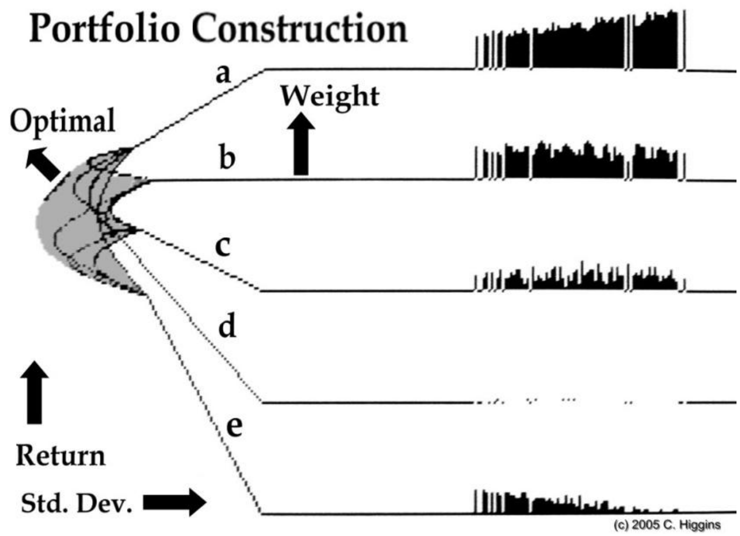

Fig. 3 Portfolio Weightings in Risk-Return Space

Thus one would consider an Efficient choice:

10) $E = fR_p - \sigma_p^2$

where f is a fractional weighting which would determine the slope of tangency on the Efficient Frontier. With N = 2 and where $W_B = 1 - W_A$:

11) $E = f(W_A R_A + [1 - W_A]R_B) - W_A^2\sigma_A^2 - 2W_A(1-W_A)\sigma_{AB} - (1-W_A)^2\sigma_B^2$

$= fW_A R_A + fR_B - fW_A R_B - W_A^2\sigma_A^2 - 2W_A\sigma_{AB} + 2W_A^2\sigma_{AB} - \sigma_B^2 + 2W_A\sigma_B^2 - W_A^2\sigma_B^2$

12) $\partial E/\partial W_A = f(R_A - R_B) - 2W_A\sigma_A^2 - 2\sigma_{AB} + 4W_A\sigma_{AB} + 2\sigma_B^2 - 2W_A\sigma_B^2 = 0$

$= f(R_A - R_B)/2 - W_A\sigma_A^2 - \sigma_{AB} + 2W_A\sigma_{AB} + \sigma_B^2 - W_A\sigma_B^2 = 0$

$= W_A(-\sigma_A^2 + 2\sigma_{AB} - \sigma_B^2) + f(R_A - R_B)/2 - \sigma_{AB} + \sigma_B^2 = 0$

13) $W_A(-\sigma_A^2 + 2\sigma_{AB} - \sigma_B^2) = \sigma_{AB} - \sigma_B^2 - f(R_A - R_B)/2$

$W_A(\sigma_A^2 - 2\sigma_{AB} + \sigma_B^2) = f(R_A - R_B)/2 + \sigma_B^2 - \sigma_{AB}$

thus:

14) $W_A = (f[R_A-R_B]/2 + \sigma_B^2 - \sigma_{AB})/(\sigma_A^2 - 2\sigma_{AB} + \sigma_B^2)$

Which is the familiar minimum portfolio variance weighting computation (see Elton, Gruber, Brown, and Goetzmann) but here including a fractional loading for the differential returns of the two securities? Consider a portfolio of two securities where A has a return of 12 percent and standard deviation of 8 and B has a return of 4 percent and standard deviation of 2 with a correlation of -.5 while recalling that $\sigma_{ij} \equiv \varrho_{ij}\sigma_i\sigma_j$. A minimum variance weighting for W_A would be:

$(2^2 - 2x8x[-.5])/(2^2 - 2x2x8x[-.5] + 8^2) = (4 + 8)/(4 + 16 + 64) = 1/7 = .143$ and $W_B = 1 - .143 = .857$. Any inclusion of $f[R_A-R_B]/2$ would increase the weighting of W_A; if f = 1 then $W_A = ([12-4]/2 + 12)/84 = 4/21 = .190$.

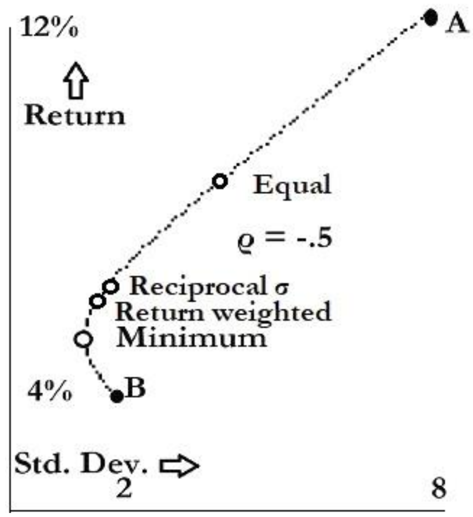

Fig. 4 The Various Portfolio Weightings

With different available weightings for portfolios, it seems appropriate to contrast them here side by side. They were equal weightings $W_i = 1/N$ with an eye toward diversifying away idiosyncratic risk, weightings which have equal impacts of risk $W_i = (1/\sigma_i)/_k\Sigma^N(1/\sigma_k)$, weightings for a minimum variance portfolio which may include an otherwise undesirable security $W_A = (\sigma_B^2-\sigma_{AB})/(\sigma_A^2-2\sigma_{AB}+\sigma_B^2)$, and may consider risk and return $W_A = (f[R_A-R_B]/2+\sigma_B^2-\sigma_{AB})/(\sigma_A^2-2\sigma_{AB}+\sigma_B^2)$.

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