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Determining Discrete Amounts of Diversification

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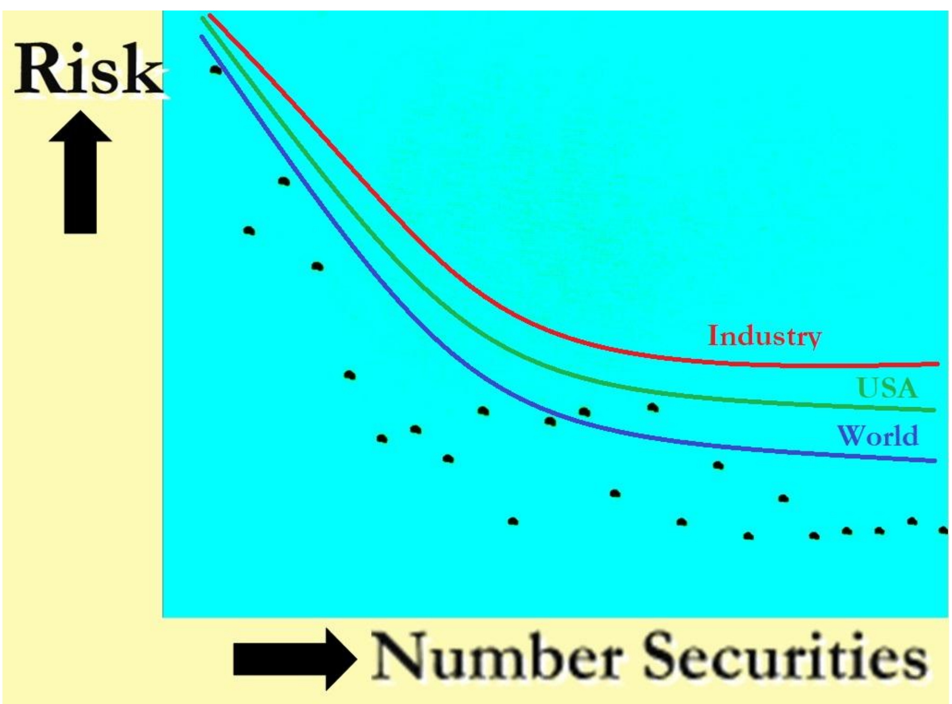
Abstract

While many demonstrations exist toward the merits of diversification, be they graphical or mathematical, a discrete finite set of measurements would be useful and also of interest.

Keywords: Diversification, portfolio

Whether it's your finance professor or your mom reminding you to diversify, it is a very good admonition. Generally it starts with the idea to not put all of your eggs in one basket in order to minimize risk and noting that minimizing risk is advisable because people often prefer to avoid losses more than a chance to make an equal gain. Whether descriptive with graphics or theoretical with mathematical abstractions the idea can be grasped but still leaves one without specificity.

The graphical argument is made with risk usually measured as standard deviation of a portfolio in the vertical and number of randomly chosen securities in the horizontal:



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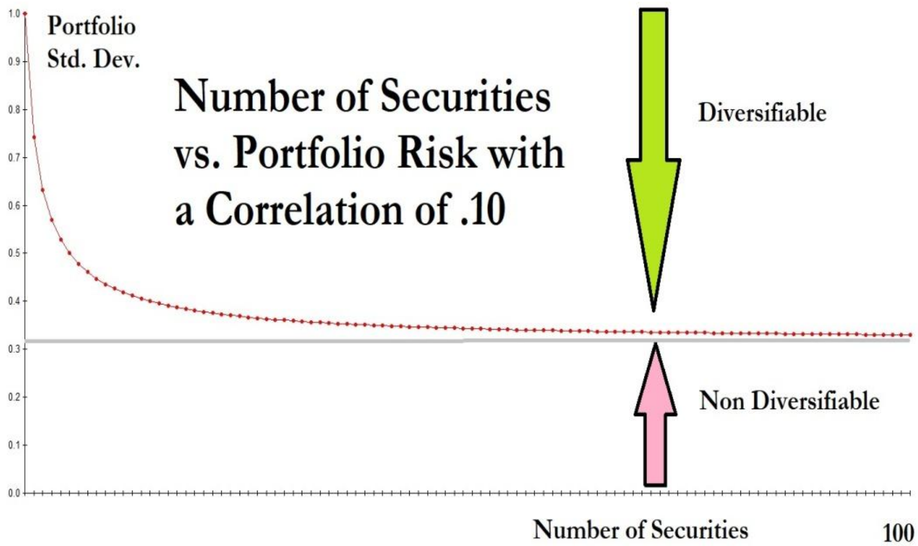
Unlike other fields of endeavor which use statistics the asymptote here does not approach zero--the other fields using statistics often seek to have a covariance that is near zero using say double blind measurements. In finance there remains a systematic market covariant risk that is positive and cannot be diversified away.



The covariance equals $\varrho_{ab}\sigma_a\sigma_b$ where ϱ_{ab} is the correlation coefficient between a and b and where σ_a and σ_b are the respective standard deviations of a and b. The correlation coefficient of the average security has often been estimated to be about .10 with of course greater variations among individual securities; see Elton and Gruber et al. [2014].

A theoretic mathematical approach to diversification notes that the portfolio variance is composed of a square matrix from $\sigma_P^2 = {}_{i=1}\Sigma^N{}_{j=1}\Sigma^N w_i w_j \sigma_{ij}$ where there are N securities, w is the weight in security i or j noting that ${}_{i=1}\Sigma^N w_i = 1$, and σ_{ij} is the covariance between i and j. The return to the portfolio is $R_P = {}_{i=1}\Sigma^N w_i R_i$. The argument proceeds with letting w = 1/N and noting that there are N variances when i = j or $\sigma_i^2 = \sigma_{ii}$; see the following graphic:

$$
\begin{matrix}
\sigma_{11} & \sigma_{12} & \sigma_{13} & \ldots & \sigma_{1n} \\
\sigma_{21} & \sigma_{22} & \sigma_{23} & \ldots & \sigma_{2n} \\
\sigma_{31} & \sigma_{32} & \sigma_{33} & \ldots & \sigma_{3n} \\
\ldots & \ldots & \ldots & \ldots & \ldots \\
\sigma_{n1} & \sigma_{n2} & \sigma_{n3} & \ldots & \sigma_{nn}
\end{matrix}
\qquad
\begin{matrix}
\sigma_1^2 & \sigma_{12} & \sigma_{13} & \ldots & \sigma_{1n} \\
\sigma_{21} & \sigma_2^2 & \sigma_{23} & \ldots & \sigma_{2n} \\
\sigma_{31} & \sigma_{32} & \sigma_3^2 & \ldots & \sigma_{3n} \\
\ldots & \ldots & \ldots & \ldots & \ldots \\
\sigma_{n1} & \sigma_{n2} & \sigma_{n3} & \ldots & \sigma_n^2
\end{matrix}
$$

Thus there are N^2-N non unique covariances and the portfolio variance becomes $N/N^2\sigma_i^2 + N(N-1)/N^2\sigma_{ij}$ or $1/N\sigma_i^2 + (N-1)/N\sigma_{ij}$. When N becomes large approaching infinity the first variance or idiosyncratic term approaches zero and serves as another argument toward diversification.

In an attempt toward a finite discrete measurement an Excel spreadsheet of increasing number of securities with the computed portfolio standard deviation (square root of the variance) using a correlation coefficient of .10 and assuming normalized standard deviations σ_i of 1. Given N variances and N^2-N non unique covariances the latter was multiplied by the correlation coefficient of .10 and the two computations were summed to give the portfolio variance which then provided its standard deviation.

As the number of securities approached 100, the total portfolio variance approached .109 from the variance component of .01 plus the covariance component of .99 times .1. Thus the asymptote in standard deviation terms becomes .330. Therefore the diversifiable risk was the difference between no diversification here 1 and .33 which equals .67. The percentage of remaining diversifiable risk was presented as the proportion that could be additionally diversified away. Here's the set of computations:

	A	B	C	D	E	F	G	H
1	No.	Var	Cov	.1*Cov	Port	SD	Div	% Div
2	1	1	0	0	1	1	0.67	100
3	2	0.5	0.5	0.05	0.55	0.742	0.412	61.44
4	3	0.333	0.667	0.067	0.4	0.632	0.302	45.14
5	4	0.25	0.75	0.075	0.325	0.57	0.24	35.83
6	5	0.2	0.8	0.08	0.28	0.529	0.199	29.72
7	6	0.167	0.833	0.083	0.25	0.5	0.17	25.37
8	7	0.143	0.857	0.086	0.229	0.478	0.148	22.1
9	8	0.125	0.875	0.088	0.213	0.461	0.131	19.55
10	9	0.111	0.889	0.089	0.2	0.447	0.117	17.49
11	10	0.1	0.9	0.09	0.19	0.436	0.106	15.8
12	11	0.091	0.909	0.091	0.182	0.426	0.096	14.39
13	12	0.083	0.917	0.092	0.175	0.418	0.088	13.18
14	13	0.077	0.923	0.092	0.169	0.411	0.081	12.15
15	14	0.071	0.929	0.093	0.164	0.405	0.075	11.24
16	15	0.067	0.933	0.093	0.16	0.4	0.07	10.45
17	16	0.063	0.938	0.094	0.156	0.395	0.065	9.744
18	17	0.059	0.941	0.094	0.153	0.391	0.061	9.116
19	18	0.056	0.944	0.094	0.15	0.387	0.057	8.552
20	19	0.053	0.947	0.095	0.147	0.384	0.054	8.043
21	20	0.05	0.95	0.095	0.145	0.381	0.051	7.58
22	21	0.048	0.952	0.095	0.143	0.378	0.048	7.159
23	22	0.045	0.955	0.095	0.141	0.375	0.045	6.773
24	23	0.043	0.957	0.096	0.139	0.373	0.043	6.418
25	24	0.042	0.958	0.096	0.138	0.371	0.041	6.091
26	25	0.04	0.96	0.096	0.136	0.369	0.039	5.788
27	26	0.038	0.962	0.096	0.135	0.367	0.037	5.507
28	27	0.037	0.963	0.096	0.133	0.365	0.035	5.246
29	28	0.036	0.964	0.096	0.132	0.364	0.034	5.002
30	29	0.034	0.966	0.097	0.131	0.362	0.032	4.774

To highlight the findings and focus upon the goals herein, here below are the more relevant number of securities in an equally weighted random chosen portfolio with an average correlation coefficient of .1:

Securities	Std. Deviation	% To Diversify
1	1.00	100
2	.74	61
3	.63	45
4	.57	36
5	.53	30
6	.50	25
7	.48	22
8	.46	20
9	.45	17
10	.44	16
11	.43	14
12	.42	13
15	.40	10
20	.38	8
25	.37	6
29	.36	5

The last set of figures is presented as a confirmation in that Elton & Gruber et al. noted that 95 percent of the diversifiable risk may be avoided with 29 securities in a portfolio of randomly chosen securities. Here $\{(1/N+.1*[N-1]/N)^{1/2}-.33\}/.67$ was the net computation approaching .33 so it's a no surprise that a rule of thumb is that the number of securities multiplied by the percentage remaining to be diversified equals about 150.

Of course given a desired amount of diversification the number of requisite randomly chosen securities can be determined. Given that the undiversified risk U equals $(1/N+\varrho[N-1]/N)^{1/2}$ with normalized standard deviations which can be achieved with weightings determined from the reciprocals of the standard deviations (see Higgins [2019]). Therefore U^2 equals $1/N+\varrho(N-1)/N$ and thus $NU^2 = 1+\varrho(N-1)$ which transforms into $NU^2 = 1+\varrho N-\varrho$ which follows into $NU^2-\varrho N = 1-\varrho$ and results in $N = (1-\varrho)/(U^2-\varrho)$. Note this requires that $U^2>\varrho$ or $U>\varrho^{1/2}$. Given here we're using ϱ =.1 which requires that U be greater than .316. Thus the following number of securities results:

Undiversified risk	Computation	Number of securities
1	(1-.1)/(1*1-.1)	1
.75	(1-.1)/(.75*.75-.1)	2
.667	(1-.1)/(.667*.667-.1)	3
.5	(1-.1)/(.5*.5-.1)	6
.333	(1-.1)/(.333*.333-.1)	81

References

Elton, EdwinJ., Martin J. Gruber, Stephen J. Brown, and William N. Goetzmann *Modern Portfolio Theory and Investment Analysis* [2014] 9th ed., Wiley

Higgins, Charles "Toward a Different Portfolio Weighting and Diversification" *Journal of Finance and Bank Management* Volume 7, Number 1 [June 2019] pp. 17-20.